Universidad Simón Bolívar
Departamento de Matemáticas
Puras y Aplicadas

Matemáticas 2 (MA1112)
Intensivo 2024
Segundo Parcial
35 puntos

Nombre: ____________________________

N° Carnet: ____________________________

Justifique sus Respuestas

1. **(2 puntos c/u)** Resuelva las siguientes ecuaciones:

   a. $\displaystyle\int_{1/3}^{x} \frac{1}{t}\,\mathrm{d}t = 2\int_{1}^{x} \frac{1}{t}\,\mathrm{d}t$, si $x > 0$

   b. $\ln(x^2+1) - \ln(x^4+x^2) + \ln(x^3) = 1$

2. **(3 puntos)** Determine la ecuación de la recta tangente a la función $y = (\cos x)^{\operatorname{sen} x}$ en el punto $(0, 1)$.

3. **(5 puntos c/u)** Calcule las siguientes integrales:

   a. $\displaystyle\int_{0}^{\ln(2)} \sqrt{e^x - 1}\,\mathrm{d}x$

   b. $\displaystyle\int \frac{1}{x\ln(x)\sqrt{\ln^2(x) - 1}}\,\mathrm{d}x$

4. **(6 puntos c/u)** Calcule las siguientes integrales:

   a. $\displaystyle\int x\cos^2(x)\operatorname{sen}(x)\,\mathrm{d}x$

   b. $\displaystyle\int_{0}^{\pi} \frac{\pi x - 1}{\sqrt{x^2 + \pi^2}}\,\mathrm{d}x$

   c. $\displaystyle\int x \operatorname{arc\,sen}(x)\,\mathrm{d}x$

SOLUCIONES

1. Resuelva las siguientes ecuaciones:

a. **(2 puntos)** $\displaystyle\int_{1/3}^{x} \frac{1}{t}\,\mathrm{d}t = 2\int_{1}^{x} \frac{1}{t}\,\mathrm{d}t$, si $x > 0$

$$\int_{1/3}^{x} \frac{1}{t}\,\mathrm{d}t = \ln(x) - \ln\left(\frac{1}{3}\right) = \ln(x) - \cancelto{0}{\ln(1)} + \ln(3) = \ln(x) + \ln(3)$$

$$\int_{1}^{x} \frac{1}{t}\,\mathrm{d}t = \ln(x)$$

Así,

$$\int_{1/3}^{x} \frac{1}{t}\,\mathrm{d}t = 2\int_{1}^{x} \frac{1}{t}\,\mathrm{d}t \Rightarrow \ln(x) + \ln(3) = 2\ln(x) \Rightarrow \ln(x) = \ln(3) \Rightarrow x = 3$$

Solución 1.a

$x = 3$

b. **(2 puntos)** $\ln(x^2+1) - \ln(x^4+x^2) + \ln(x^3) = 1$

El dominio de posibles soluciones para esta ecuación ($\mathrm{D_s}$) son todos los valores de x que cumplen $x^2+1 > 0$, $x^4+x^2 > 0$ y $x^3 > 0$, es decir, $\mathrm{D_s} = x \in (0, \infty)$

De aquí,

$$\ln(x^2+1) - \ln(x^4+x^2) + \ln(x^3) = 1 \Rightarrow \ln\left(\frac{x^2+1}{x^4+x^2}\cdot x^3\right) = \ln(e) \Rightarrow \ln\left(\frac{x^3(x^2+1)}{x^2(x^2+1)}\right) = \ln(e)$$

$$\Rightarrow \ln(x) = \ln(e) \Rightarrow x = e$$

Solución 1.b

$x = e$

2. **(3 puntos)** Determine la ecuación de la recta tangente a la función $y = (\cos x)^{\operatorname{sen} x}$ en el punto $(0, 1)$.

$$y = (\cos x)^{\operatorname{sen} x} = e^{\ln\left((\cos x)^{\operatorname{sen} x}\right)} = e^{\operatorname{sen} x \ln(\cos x)}$$

$$y' = \left(e^{\operatorname{sen} x \ln(\cos x)}\right)' = e^{\operatorname{sen} x \ln(\cos x)} \cdot \left(\cos x \ln(\cos x) + \operatorname{sen} x \cdot \frac{-\operatorname{sen} x}{\cos x}\right) =$$

$$= (\cos x)^{\operatorname{sen} x} \cdot \left(\cos x \ln(\cos x) - \frac{\operatorname{sen}^2 x}{\cos x}\right)$$

En $x = 0$:

$$y'\Big|_{x=0} = (\cos 0)^{\operatorname{sen} 0} \cdot \left(\cos 0 \ln(\cos 0) - \frac{\operatorname{sen}^2 0}{\cos 0}\right) = (1)^0 \cdot \left(1 \ln(1) - \frac{0}{1}\right) = 1 \cdot (0 - 0) = 0$$

Como la pendiente en la recta tangente en el punto $(0, 1)$ es 0, la recta tangente es una constante, así:

Solución 2

La ec. de la recta tangente es: $y = 1$

3. **(5 puntos c/u)** Calcule las siguientes integrales:

a. $\displaystyle\int_0^{\ln(2)} \sqrt{e^x - 1}\,\mathrm{d}x$

Cambio de variable:

$$\begin{aligned} &e^x - 1 = u^2 \\ &e^x = u^2 + 1 && x = \ln(2) \Rightarrow u = 1 \\ &x = \ln(u^2 + 1) && x = 0 \Rightarrow u = 0 \\ &\mathrm{d}x = \frac{2u}{u^2 + 1}\mathrm{d}u \end{aligned}$$

Nos queda:

$$\int_0^{\ln(2)} \sqrt{e^x - 1}\,\mathrm{d}x = \int_0^1 \sqrt{u^2} \cdot \frac{2u}{u^2 + 1}\mathrm{d}u = 2\int_0^1 \frac{u^2}{u^2 + 1}\mathrm{d}u = 2\int_0^1 \frac{u^2 + 1 - 1}{u^2 + 1}\mathrm{d}u = 2\int_0^1 \left(\frac{u^2 + 1}{u^2 + 1} - \frac{1}{u^2 + 1}\right)\mathrm{d}u =$$
$$= 2\int_0^1 \left(1 - \frac{1}{u^2 + 1}\right)\mathrm{d}u = 2(u - \arctan(u)\Big|_0^1 = 2(1 - \arctan(1)) = 2\left(1 - \frac{\pi}{4}\right) = 2 - \frac{\pi}{2}$$

Solución 3.a

$$\int_0^{\ln(2)} \sqrt{e^x - 1}\,\mathrm{d}x = 2 - \frac{\pi}{2}$$

b. $\displaystyle\int \frac{1}{x \ln(x)\sqrt{\ln^2(x) - 1}}\mathrm{d}x$

Cambio de variable:

$$\begin{aligned} &u = \ln(x) \\ &\mathrm{d}u = \frac{\mathrm{d}x}{x} \end{aligned}$$

Nos queda:

$$\int \frac{1}{x\ln(x)\sqrt{\ln^2(x)-1}}\,\mathrm{d}x = \int \frac{\mathrm{d}u}{u\sqrt{u^2-1}}$$

Sustitución trigonométrica:

$$u = \sec\Theta$$
$$\mathrm{d}u = \sec\Theta\tan\Theta\,\mathrm{d}\Theta$$
$$\frac{\mathrm{d}u}{u} = \tan\Theta\,\mathrm{d}\Theta$$

Nos queda:

$$\int \frac{\mathrm{d}u}{u\sqrt{u^2-1}} = \int \frac{\tan\Theta}{\sqrt{\sec^2\Theta-1}}\,\mathrm{d}\Theta = \int \frac{\tan\Theta}{\sqrt{\tan^2\Theta}}\,\mathrm{d}\Theta = \int \frac{\tan\Theta}{\tan\Theta}\,\mathrm{d}\Theta = \int \mathrm{d}\Theta = \Theta + C =$$
$$= \sec^{-1} u + C = \sec^{-1}(\ln x) + C$$

Solución 1.b

$$\int \frac{1}{x\ln(x)\sqrt{\ln^2(x)-1}}\,\mathrm{d}x = \sec^{-1}(\ln x) + C$$

4. **(6 puntos c/u)** Calcule las siguientes integrales:

a. $\int x\cos^2(x)\,\mathrm{sen}(x)\,\mathrm{d}x$

Cambio de variable:

$$u = \cos x$$
$$x = \arccos u$$
$$-\mathrm{d}u = \mathrm{sen}(x)\,\mathrm{d}x$$

Nos queda:

$$\int x\cos^2(x)\,\mathrm{sen}(x)\,\mathrm{d}x = -\int u^2\arccos(u)\,\mathrm{d}u$$

Integración por partes:

$$f(u) = \arccos u \quad\longrightarrow\quad f'(u) = \frac{-1}{\sqrt{1-u^2}}$$
$$g'(u) = u^2 \qquad\qquad g(u) = \frac{1}{3}u^3$$

Nos queda:

$$-\int u^2\arccos(u)\,\mathrm{d}u = -\frac{1}{3}u^3\arccos u + \underbrace{\frac{1}{3}\int \frac{-u^3}{\sqrt{1-u^2}}\,\mathrm{d}u}_{\mathrm{I}}$$

$$\mathrm{I} = \frac{1}{3}\int \frac{-u^3}{\sqrt{1-u^2}}\,\mathrm{d}u = \frac{1}{6}\int \frac{u^2}{\sqrt{1-u^2}}(-2u\,\mathrm{d}u) = -\frac{1}{6}\int \frac{-u^2}{\sqrt{1-u^2}}(-2u\,\mathrm{d}u) = -\frac{1}{6}\int \frac{1-u^2-1}{\sqrt{1-u^2}}(-2u\,\mathrm{d}u)$$

Cambio de variable:

$$t^2 = 1-u^2$$
$$2t\,\mathrm{d}t = -2u\,\mathrm{d}u$$

Nos queda:

$$\mathrm{I} = -\frac{1}{6}\int \frac{t^2-1}{t}(2t\,\mathrm{d}t) = \frac{1}{3}\int (1-t^2)\,\mathrm{d}t = \frac{1}{3}(t - \frac{1}{3}t^3) + C = \frac{1}{3}\sqrt{1-u^2} - \frac{1}{9}(\sqrt{1-u^2})^3 + C$$

Volviendo a la integral principal:

$$-\int u^2 \arccos(u)\,\mathrm{d}u = -\frac{1}{3}u^3\arccos u + \mathrm{I} = -\frac{1}{3}u^3\arccos u + \frac{1}{3}\sqrt{1-u^2} - \frac{1}{9}(\sqrt{1-u^2})^3 + C$$

$$\Rightarrow \int x\cos^2(x)\operatorname{sen}(x)\,\mathrm{d}x = -\frac{1}{3}x\cos^3(x) + \frac{1}{3}\sqrt{1-\cos^2(x)} - \frac{1}{9}(\sqrt{1-\cos^2(x)})^3 + C =$$
$$= -\frac{1}{3}x\cos^3(x) + \frac{1}{3}\sqrt{\operatorname{sen}^2(x)} - \frac{1}{9}(\sqrt{\operatorname{sen}^2(x)})^3 + C = -\frac{1}{3}x\cos^3(x) + \frac{1}{3}\operatorname{sen} x - \frac{1}{9}\operatorname{sen}^3(x) + C$$

Solución 4.a

$$\int x\cos^2(x)\operatorname{sen}(x)\,\mathrm{d}x = \frac{1}{3}\operatorname{sen} x - \frac{1}{3}x\cos^3(x) - \frac{1}{9}\operatorname{sen}^3(x) + C$$

b. $$\int_0^{\pi} \frac{\pi x - 1}{\sqrt{x^2+\pi^2}}\,\mathrm{d}x = \underbrace{\int_0^{\pi} \frac{\pi x}{\sqrt{x^2+\pi^2}}\,\mathrm{d}x}_{\mathrm{I}} - \underbrace{\int_0^{\pi} \frac{1}{\sqrt{x^2+\pi^2}}\,\mathrm{d}x}_{\mathrm{II}}$$

$$\mathrm{I} = \int_0^{\pi} \frac{\pi x}{\sqrt{x^2+\pi^2}}\,\mathrm{d}x = \pi\int_0^{\pi} \frac{2x}{2\sqrt{x^2+\pi^2}}\,\mathrm{d}x = \pi\sqrt{x^2+\pi^2}\Big|_0^{\pi} = \pi\sqrt{2\pi^2} - \pi\sqrt{\pi^2} = \pi^2\sqrt{2} - \pi^2 =$$
$$= \pi^2(\sqrt{2}-1)$$

Para II, empezamos con una sustitución trigonométrica:

$$x = \pi\tan\Theta \qquad x = \pi \Rightarrow \Theta = \frac{\pi}{4}$$
$$\mathrm{d}x = \pi\sec^2\Theta\,\mathrm{d}\Theta \qquad x = 0 \Rightarrow \Theta = 0$$

$$\int_0^{\pi} \frac{1}{\sqrt{x^2+\pi^2}}\,\mathrm{d}x = \int_0^{\pi/4} \frac{\pi\sec^2\Theta}{\sqrt{\pi^2\tan^2\Theta+\pi^2}}\,\mathrm{d}\Theta = \int_0^{\pi/4} \frac{\pi\sec^2\Theta}{\pi\sqrt{\tan^2\Theta+1}}\,\mathrm{d}\Theta = \int_0^{\pi/4} \frac{\sec^2\Theta}{\sqrt{\sec^2\Theta}}\,\mathrm{d}\Theta =$$
$$= \int_0^{\pi/4} \frac{\sec^2\Theta}{\sec\Theta}\,\mathrm{d}\Theta = \int_0^{\pi/4} \sec\Theta\,\mathrm{d}\Theta = \ln(\sec\Theta + \tan\Theta)\Big|_0^{\pi/4} = \ln\left(\sec\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right)\right) - \ln(\sec(0) + \tan(0)) =$$
$$= \ln\left(\sqrt{2}+1\right) - \ln(1) = \ln\left(\sqrt{2}+1\right)$$

Luego,

$$\int_0^{\pi} \frac{\pi x - 1}{\sqrt{x^2+\pi^2}}\,\mathrm{d}x = \mathrm{I} - \mathrm{II} = \pi^2(\sqrt{2}-1) - \ln\left(\sqrt{2}+1\right)$$

Solución 4.b

$$\int_0^{\pi} \frac{\pi x - 1}{\sqrt{x^2+\pi^2}}\,\mathrm{d}x = \pi^2(\sqrt{2}-1) - \ln\left(\sqrt{2}+1\right)$$

c. $\displaystyle\int x \operatorname{arc\,sen}(x)\,\mathrm{d}x$

Integración por partes:

$$f(x) = \operatorname{arc\,sen} x \quad \longrightarrow \quad f'(x) = \frac{1}{\sqrt{1-x^2}}$$
$$g'(x) = x \qquad\qquad g(x) = \frac{1}{2}x^2$$

Nos queda:

$$\int x \operatorname{arc\,sen}(x)\,\mathrm{d}x = \frac{1}{2}x^2 \operatorname{arc\,sen} x \underbrace{-\frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,\mathrm{d}x}_{\mathrm{I}}$$

Sustitución trigonométrica para I:

$x = \operatorname{sen}\Theta$
$\mathrm{d}x = \cos\Theta\,\mathrm{d}\Theta$

Nos queda:

$$-\frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,\mathrm{d}x = -\frac{1}{2}\int \frac{\operatorname{sen}^2\Theta\cos\Theta}{\sqrt{1-\operatorname{sen}^2\Theta}}\,\mathrm{d}\Theta = -\frac{1}{2}\int \frac{\operatorname{sen}^2\Theta\cos\Theta}{\sqrt{\cos^2\Theta}}\,\mathrm{d}\Theta = -\frac{1}{2}\int \frac{\operatorname{sen}^2\Theta\cos\Theta}{\cos\Theta}\,\mathrm{d}\Theta =$$
$$= -\frac{1}{2}\int \operatorname{sen}^2\Theta\,\mathrm{d}\Theta = -\frac{1}{2}\int \left(\frac{1-\cos(2\Theta)}{2}\right)\mathrm{d}\Theta = -\frac{1}{4}\int \mathrm{d}\Theta + \frac{1}{4}\int \cos(2\Theta)\,\mathrm{d}\Theta = -\frac{1}{4}\Theta + \frac{1}{8}\int 2\cos(2\Theta)\,\mathrm{d}\Theta =$$
$$= -\frac{1}{4}\Theta + \frac{1}{8}\operatorname{sen}(2\Theta) + C = -\frac{1}{4}\Theta + \frac{1}{4}\operatorname{sen}\Theta\cos\Theta + C = -\frac{1}{4}\Theta + \frac{1}{4}\operatorname{sen}\Theta\sqrt{1-\operatorname{sen}^2\Theta} + C =$$
$$= -\frac{1}{4}\operatorname{arc\,sen}(x) + \frac{1}{4}x\sqrt{1-x^2} + C$$

Volviendo a la integral principal:

$$\int x \operatorname{arc\,sen}(x)\,\mathrm{d}x = \frac{1}{2}x^2 \operatorname{arc\,sen} x + \mathrm{I} = \frac{1}{2}x^2 \operatorname{arc\,sen} x - \frac{1}{4}\operatorname{arc\,sen}(x) + \frac{1}{4}x\sqrt{1-x^2} + C$$

Solución 4.c

$$\int x \operatorname{arc\,sen}(x)\,\mathrm{d}x = \frac{1}{2}x^2 \operatorname{arc\,sen} x - \frac{1}{4}\operatorname{arc\,sen}(x) + \frac{1}{4}x\sqrt{1-x^2} + C$$

Este parcial fue digitalizado en LaTeX por **Daniel Quijada** para **GECOUSB**

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Cualquier error en la resolución de los ejercicios, notificar a **20-10518@usb.ve**